T-Duality and Non-Local Supersymmetries

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ABSTRACT

We study the non-localization of extended worldsheet supersymmetry under T-duality, when the associated complex structure depends on the coordinate with respect to which duality is performed. First, the canonical transformation which implements T-duality is generalized to the supersymmetric non-linear σ-models. Then, we obtain the non-local object which replaces the complex structure in the dual theory and write down the condition it should satisfy so that the dual action is invariant under the non-local supersymmetry. For the target space, this implies that the supersymmetry transformation parameter is a non-local spinor. The analogue of the Killing equation for this non-local spinor is obtained. It is argued that in the target space, the supersymmetry is no longer realized in the standard way. The string theoretic origin of this phenomenon is briefly discussed.

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1   Introduction

It is known that one should expect certain non-local effects to appear in an effective field theory based on string theory [1]. This clearly has to do with the fact that, unlike a point particle, the string is not a dimensionless object. Such effects in the low-energy theory have not yet been studied in detail, probably due to the absence of compulsive evidence for their importance. However, there is significant evidence that they do appear in connection with important issues such as T-duality and supersymmetry [2, 3, 4, 5]. In this paper we will report on an investigation of the issue of non-localization of extended worldsheet supersymmetry and the associated target space supersymmetry under a T-duality transformation.

A given conformal field theory may have different target space realizations which are related to each other by a T-duality transformation [7, 8, 9, 10, 11, 12]. The mechanism by which a T-duality transformation gives rise to a non-locality (in the target-space sense) is most transparent when the duality transformations are formulated as canonical transformations in the worldsheet theory [4, 13, 14, 15]. In this approach, the coordinate (say θ) with respect to which duality is performed and the corresponding coordinate in the dual theory (say $\widetilde{\theta}$) are non-local functions of each other. The non-locality is a consequence of an integration over the string length parameter which appears in the relation between θ and $\widetilde{\theta}$. As a result, any θ-dependent quantity in one theory becomes a non-local function of the corresponding coordinate in the dual theory. This effect, which is also accompanied by the interchange between momentum modes (*local*) and winding modes (*non-local*), is solely due the extended nature of the string. Since a duality transformation with respect to the coordinate θ is performed only when the massless background fields are independent of θ, the non-localities will not show up unless we go beyond this set of fields. An example is a WZNW model based on a group G and the corresponding symmetry currents. In this case, the group G may have a non-local realization in the dual theory [2, 3, 4].

A more interesting situation, in which non-local effects show up after a duality transformation, arises when the worldsheet theory has an extended supersymmetry [16, 17, 18, 19, 20]. It is known that if a complex structure associated with an extended supersymmetry on the worldsheet, does not have a dependence on the coordinate θ, then in the dual model the extended supersymmetry is realized in the usual way [21, 22]. However, it was noticed in [6] that in certain examples supersymmetry is not preserved under a duality transformation. It turns out that in all these examples, the complex structure associated with the supersymmetry under consideration depends on the coordinate θ with respect to which duality is performed. Then, from the discussion above, it follows that in the dual theory the complex structure is replaced by a non-local object. A prescription for obtaining this non-local object was suggested in [5]. This phenomenon implies that, in such a situation, the extended supersymmetry of the dual theory, though still present, is realized non-locally. In particular, the relation between supersymmetry and target space geometry is modified. Since worldsheet supersymmetry is intimately connected with target space supersymmetry, this non-locality

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is also expected to have implications for the latter. However, one should keep in mind that as string theories, the dual models are, nevertheless, physically equivalent.

The mechanism by which the non-locality arises is not specific to duality transformations. It is, in fact, common to all non-trivial $O(d, d)$ transformations which generically relate physically inequivalent background field configurations and which contain T-duality as a discrete subgroup.

In this paper, we investigate the phenomenon of non-localization of supersymmetry under a T-duality transformation. The paper is organized as follows: In section 2, we describe our conventions and then generalize the canonical transformation which implements T-duality from bosonic to supersymmetric non-linear σ-models. This canonical transformation can be written in the superfield notation. In section 3, we consider a non-linear σ-model with extended supersymmetry on the worldsheet such that the associated complex structure is θ-dependent. We obtain the non-local object which, in the dual theory, replaces the θ-dependent complex structure. We then obtain the conditions on this non-local object which are analogous to the covariant constancy of complex structure in the standard realization of the extended supersymmetry. In section 4, we show that, in the dual theory, the Killing spinor associated with the target space supersymmetry is also replaced by a non-local spinor. We obtain the analogue of the Killing equation which this non-local spinor satisfies and discuss its consequence for the realization of non-local target space supersymmetry. At the end, we briefly discuss the string theoretic origin of the non-locality when θ is a compact coordinate. In section 5, we summarize our results and point out an example where S-duality seems to be incompatible with the standard realization of worldsheet supersymmetry.

2   T-Duality as a Canonical Transformation in Supersymmetric Theories

In this section, we generalize the method of realizing a T-duality transformation by a canonical transformation on the worldsheet to include $N = 1$ supersymmetric non-linear σ-models.

We start with the non-linear σ-model with $N = 1$ supersymmetry on the worldsheet. Following the conventions of [23], in the component notation the action defining this model takes the form

$$
\begin{aligned}
S \;=\; & \frac{1}{2} \int d^2\sigma [(G_{MN} + B_{MN})\partial_+ X^M \partial_- X^N \\
& - \; i\psi_+^M G_{MN}(\delta_K^N \partial_- + \Omega_{LK}^{+N}\partial_- X^L)\psi_+^K - i\psi_-^M G_{MN}(\delta_K^N \partial_+ + \Omega_{LK}^{-N}\partial_+ X^L)\psi_-^K \\
& + \; \frac{1}{2}\psi_+^M \psi_+^N \psi_-^K \psi_-^L R_{MNKL}(\Omega^-)].
\end{aligned}
\tag{1}
$$

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Here, $\Omega^\pm$ are the torsionful connections given by $\Omega_{MN}^{\pm K} = \Gamma_{MN}^K \pm \frac{1}{2} G^{KL} H_{LMN}$, where Γ_{MN}^K is the Christoffel symbol and H_{MNK} is the torsion tensor given by $H_{MNK} = 3\partial_{[M} B_{NK]}$; $R_{MNKL}(\Omega^\pm)$ are the curvature tensors corresponding to the torsionful connections and satisfy the property $R_{MNKL}(\Omega^-) = R_{KLMN}(\Omega^+)$. The above action has a default $N = 1$ supersymmetry which has independent action on the left-moving and the right-moving chiral sectors of the theory. The transformations of the fields under this $(1,1)$ supersymmetry are given by

$$\delta_\mp X^M = \pm i\epsilon_\mp \psi_\pm^M \tag{2}$$

$$\delta_\mp \psi_\pm^M = \pm \partial_\pm X^M \epsilon_\mp \tag{3}$$

$$\delta_\mp \psi_\mp^M = \mp i\psi_\mp^N \epsilon_\mp \Omega_{NK}^{\pm M} \psi_\pm^K. \tag{4}$$

In the following, we describe the implementation of T-duality transformations by canonical transformations in the above $N = 1$ theory. Similar issues in the context of the chiral model have been discussed in [24]. In different contexts, T-duality in supersymmetric theories has been studied in [25].

A T-duality transformation is always performed with respect to a Killing vector field K defined by $\mathcal{L}_K G = 0$, $\mathcal{L}_K B = d\omega_{(K)}$ and $\mathcal{L}_K \Phi = 0$. Here, $\mathcal{L}_K$ denotes the Lie derivative along the vector field K and $\omega_{(K)}$ is a one-form in the target space; Φ is the dilaton field, which does not appear in (1). We choose a coordinate system $X^1 = \theta, X^{i+1} = x^i; i = 1, ..., D - 1$ such that the Killing vector takes the form $K = \partial/\partial\theta$. In this coordinate system, the background fields G and B can be chosen to be independent of θ and, under a duality transformation with respect to θ, transform to

$$\widetilde{G}_{\theta\theta} = G_{\theta\theta}^{-1},$$
$$(\widetilde{G} + \widetilde{B})_{\theta i} = -G_{\theta\theta}^{-1}(G + B)_{\theta i},$$
$$(\widetilde{G} - \widetilde{B})_{\theta i} = G_{\theta\theta}^{-1}(G - B)_{\theta i},$$
$$(\widetilde{G} + \widetilde{B})_{ij} = (G + B)_{ij} - G_{\theta\theta}^{-1}(G - B)_{\theta i}(G + B)_{\theta j}. \tag{5}$$

In many situations, as in equations (2)–(4), the field B_{MN} appears only in the torsionful connections $\Omega^\pm$ through its field strength tensor H_{MNK}. In these situations, it is convenient to rewrite the duality transformations in terms of the relevant variables which are G_{MN} and $\Omega_{NK}^{\pm M}$. To do this, let us introduce two $D \times D$ dimensional matrices $Q_\pm$ given by [1]

$$Q_\pm = \begin{pmatrix} \mp G_{\theta\theta} & \mp(G \mp B)_{\theta i} \\ 0 & 1_{D-1} \end{pmatrix}, \qquad Q_\pm^{-1} = \begin{pmatrix} \mp G_{\theta\theta}^{-1} & -G_{\theta\theta}^{-1}(G \mp B)_{\theta i} \\ 0 & 1_{D-1} \end{pmatrix}. \tag{6}$$

In terms of these, the dual metric is given by

$$\widetilde{G}^{-1} = Q_- G^{-1} Q_-^T = Q_+ G^{-1} Q_+^T, \tag{7}$$

[1] The matrices $Q_\pm$ were introduced in [22], in connection with arbitrary non-trivial $O(d,d)$ transformations. For the discrete duality subgroup, they reduce to the ones given above.

and the dual torsionful connections take the form

$$\widetilde{\Omega}_{NK}^{\pm M} = (Q_\mp^{-1})_N^{N'} (Q_\pm^{-1})_K^{K'} (Q_\mp)_{M'}^M \Omega_{N'K'}^{\pm M'} - \delta_K^i (\partial_i Q_\mp Q_\mp^{-1})_N^M. \tag{8}$$

The above equation can be obtained using the formalism in [22]. Note that though the metric transforms unambiguously, the (inverse) vielbein e, defined by $G^{-1} = e \eta e^T$, does not. However, the two possible options, $\widetilde{e}_+ = Q_+ e$ and $\widetilde{e}_- = Q_- e$, are related by a local Lorentz transformation: $\widetilde{e}_+ = \widetilde{e}_- \Lambda$. Here, Λ is given by $\Lambda = e^{-1} Q_-^{-1} Q_+ e$ and it satisfies $\Lambda \eta \Lambda^T = \eta$. Note that, in the above, we have assumed e to be θ-independent.

The transformations of the background fields (5) can be obtained in different ways depending on the method one chooses to implement duality. However, the ease with which these methods could be generalized to produce the transformation under duality of other operators in the theory, varies from method to method. From this point of view, the canonical approach seems to be the most powerful. In bosonic non-linear σ-models, the implementation of duality by a canonical transformation was first used in [13] for constant background fields and then discussed in more detail in [4]: Let p_θ denote the canonical momentum conjugate to the coordinate θ and let $\theta' = \partial \theta / \partial \sigma$. The duality transformations (5) now follow from the canonical transformation

$$\widetilde{\theta}' = -p_\theta \,, \qquad \widetilde{p}_\theta = -\theta' \,, \qquad \widetilde{x}^i = x^i \,. \tag{9}$$

It is clear from the above that the relation between θ and $\widetilde{\theta}$ is, in general, non-local.

We want to apply this procedure to the supersymmetric model defined by the action (1). The θ-independence of the background fields in this model gives rise to a conserved isometry current with components $\mathcal{I}_\pm$ given by

$$\mathcal{I}_\pm = \frac{1}{2}(G \mp B)_{\theta M} \, \partial_\pm X^M - \frac{i}{2} F_\pm \tag{10}$$

where we have used the notation

$$F_\pm \equiv \psi_\pm^M \, G_{MN} \, \Omega_{\theta L}^{\pm N} \, \psi_\pm^L = \psi_\pm^M \, \psi_\pm^j \, \partial_j (G \mp B)_{\theta M}. \tag{11}$$

The canonical momentum conjugate to θ can be obtained from (1) and is given by

$$\begin{aligned} p_\theta &= \mathcal{I}_+ + \mathcal{I}_- \\ &= G_{\theta\theta} \dot{\theta} + \frac{1}{2}(G - B)_{\theta j} \, \partial_+ x^j + \frac{1}{2}(G + B)_{\theta j} \, \partial_- x^j - \frac{i}{2}(F_+ + F_-). \end{aligned} \tag{12}$$

It is clear that in order to obtain the dual theory in the supersymmetric case, the canonical transformation (9), with p_θ given by (12), is not sufficient. It has to be accompanied by the appropriate transformations of the worldsheet fermions so that $N = 1$ supersymmetry is preserved. The required transformations of fermions can be obtained by demanding that the supersymmetry transformation equations (2)–(4) imply a similar set of equations for the dual

theory, provided the backgrounds in the two theories are still related by (5). First, consider eq. (3) which, in the canonically transformed theory, should take the form $\delta_{\mp}\widetilde{\psi}_{\pm}^{M} = \partial_{\pm}\widetilde{X}^{M}\epsilon_{\mp}$. Since the canonical transformation (9) does not affect the coordinates x^{i}, we get $\widetilde{\psi}_{+}^{i} = \psi_{+}^{i}$. For the ψ^{θ}-component, using (9) and (12) along with the duality relations (5), we obtain $\widetilde{\psi}_{\pm}^{\theta} = \mp(G \mp B)_{\theta M}\psi_{\pm}^{M}$. These transformations of fermions, which should accompany the canonical transformation (9), can be written in terms of the matrices $Q_{\pm}$ as

$$\widetilde{\psi}_{\pm}^{M} = Q_{\pm N}^{M}\psi_{\pm}^{N}. \tag{13}$$

The origin of the difference in transformations of ψ_{+}^{M} and ψ_{-}^{M} can be traced back to the interpretation of duality (and for that matter, all non-trivial $O(d,d)$ transformations) as Lorentz transformations acting independently on the two chiral sectors of the underlying conformal field theory [32]. Note that since the zero modes of worldsheet fermions are related to the Clifford algebra in the target space, the above difference may have an implication for the target space supersymmetry. However, as will be discussed in section 4, this difference can be absorbed in a redefinition of the corresponding target space spinors. The compatibility of the transformations (9) and (13) with the remaining two of the $N = 1$ supersymmetry transformation equations (2) and (4) will be discussed below.

Let us consider the relation between the original and the dual coordinates. Using (9) and (13), this can be written as

$$\partial_{\pm}\widetilde{\theta} = Q_{\pm M}^{\theta}\,\partial_{\pm}X^{M} + i\psi_{\pm}^{j}\,\partial_{j}Q_{\pm M}^{\theta}\,\psi_{\pm}^{M}. \tag{14}$$

Comparing the above equations with (10), we get $\partial_{\pm}\widetilde{\theta} = \mp 2\mathcal{I}_{\pm}$, so that

$$\widetilde{\theta} = 2\int d\sigma^{-}\mathcal{I}_{-} - 2\int d\sigma^{+}\mathcal{I}_{+}. \tag{15}$$

The above relation tells us that the dual coordinate is a non-local function of the original coordinates. However, the conservation of the isometry current, $\partial_{+}\mathcal{I}_{-} + \partial_{-}\mathcal{I}_{+} = 0$, leads to $\partial_{+}\partial_{-}\widetilde{\theta} = \partial_{-}\partial_{+}\widetilde{\theta}$. This implies that in spite of the non-local relation between θ and $\widetilde{\theta}$, on shell, the dual coordinate is well defined on the worldsheet and is a local function of the worldsheet coordinates. As a digression, notice that if $G_{\theta\theta} = 1$ and $(G + B)_{\theta i} = 0$, or equivalently if $Q_{-} = 1_{D}$, then the backgrounds are self-dual and we do not expect a non-locality to show up in the transformations. In fact in this case, $\partial_{-}\widetilde{\theta} = \partial_{-}\theta$ which has a solution $\widetilde{\theta} = \theta + f(\sigma^{+})$, where the function $f(\sigma^{+})$ is determined by $\partial_{+}\widetilde{\theta} = -2\mathcal{I}_{+}$. However, the isometry current conservation equation now takes the form $\partial_{-}(\partial_{+}\theta + 2\mathcal{I}_{+}) = 0$. This defines a chirally conserved current and the action (1) develops an invariance under σ^{+}-dependent translations of θ. This invariance can be used to eliminate $f(\sigma^{+})$ and set $\theta = \widetilde{\theta}$. This proves that a duality transformation with respect to a chiral isometry does not result in a non-locality [3, 4].

Now, we consider the compatibility of the canonical transformations (9) and (13) (or equivalently, (13) and (14)) with the remaining two of the $N = 1$ supersymmetry transformations, *i.e.* eqs. (2) and (4). First, using (8), it is easy to see that under a canonical

transformation, (4) goes over to a similar equation in the dual theory and is therefore compatible with our transformations. As for eq. (2), only the θ-component is non-trivial. Due to the non-local relation between θ and $\widetilde{\theta}$, one way to study the behaviour of this equation under a canonical transformation is to consider its derivatives with respect to $\sigma^\pm$. This is sufficient because the action is already invariant under constant shifts of θ. For the sake of clarity, let us concentrate on the equation involving ϵ_-. Here, it is again easy to show that the derivative of (2) with respect to σ^+ gives rise to a similar equation for the dual variables, i.e. $\delta_-(\partial_+\widetilde{\theta}) = \epsilon_-\partial_+\widetilde{\psi}^\theta_+$. The σ^- derivative of (2) is slightly different and after a canonical transformation leads to

$$\delta_-(\partial_-\widetilde{\theta}) = i\epsilon_-(\partial_-\widetilde{\psi}^\theta_+) - 4\epsilon_-\frac{\delta S}{\delta\psi^\theta_+}. \tag{16}$$

On shell, $\delta S/\delta\psi^\theta_+ = 0$, and the above equation reduces to the desired supersymmetry transformation for the dual theory. This completes the proof that the canonical transformation (9) accompanied by the transformations (13) of the worldsheet fermions (or equivalently transformations (13) and (14)) are compatible with the $N = 1$ supersymmetry transformations (2)–(4).

The canonical transformations (14) and (13) can be written, in a compact form, in terms of the $N = 1$ superfields Φ^M as

$$D_\pm\widetilde{\Phi}^M = Q^M_{\pm N}(\Phi)\, D_\pm\Phi^N. \tag{17}$$

The above equation holds on shell and also contains the transformation of the auxiliary field under duality which is consistent with its equation of motion.

Since the fermion couplings in (1) are entirely determined by the $N = 1$ supersymmetry, the above discussion guarantees that the canonically transformed action has the same form as the original action (1) with the backgrounds G_{MN}, B_{MN} replaced by their dual counterparts given by (5). This can be checked by going to the Hamiltonian formulation where the canonical transformation takes the form $\widetilde{H}(\widetilde{p}_\theta, \widetilde{\theta}', \widetilde{\psi}_\pm) = H(p_\theta, \theta', \psi_\pm)$. Rewritten back in terms of the Lagrangian, this gives

$$\widetilde{L}(\partial_\pm\widetilde{\theta}, \widetilde{\psi}^\theta_\pm) = L(\partial_\pm\theta, \psi^\theta_\pm) + \frac{1}{2}(\partial_+\widetilde{\theta}\,\partial_-\theta - \partial_-\widetilde{\theta}\,\partial_+\theta) \tag{18}$$

where the variables $\partial_\pm\theta, \psi^\theta_\pm$ on the right-hand side have to be expressed in terms of $\partial_\pm\widetilde{\theta}, \widetilde{\psi}^\theta_\pm$ using (14) and (13). Now it is a matter of calculation to check that the Lagrangians L and $\widetilde{L}$ have exactly the same form when the background fields G and B appearing in the two are related by (5). Going through this calculation, one can see that the fermion-dependent terms $F_\pm$ appearing in the expression for the canonical momentum p_θ contribute only to the four-fermion terms in the dual theory. Their presence is therefore necessary to reproduce the correct transformation of the curvature tensor under duality. By comparing L and $\widetilde{L}$, we can very easily obtain the following transformation equation involving the curvature tensor, which we note down for later use:

$$\widetilde{\psi}^M_+\widetilde{\psi}^N_+\widetilde{\psi}^K_-\widetilde{\psi}^L_-\widetilde{R}_{MNKL}(\widetilde{\Omega}^-) = \psi^M_+\psi^N_+\psi^K_-\psi^L_- R_{MNKL}(\Omega^-) + 2G^{-1}_{\theta\theta}F_+F_-. \tag{19}$$

Having generalized the canonical approach to T-duality to the case of supersymmetric non-linear σ-models, in the next section we turn to the issue of its effect on extended worldsheet supersymmetry.

3   T-Duality and Non-Local Extended Supersymmetry

In this section, we consider a σ-model with extended supersymmetry on the worldsheet such that the complex structure associated with the extended supersymmetry is not independent of the coordinate with respect to which duality is performed. We obtain the non-local object which replaces the complex structure in the dual theory and write down the equation which it should satisfy so that the dual action is invariant under the corresponding non-local supersymmetry transformations.

We begin with a review of the usual realization of the extended worldsheet supersymmetry in order to facilitate comparison with its non-local realization in the dual theory. If the target space manifold admits almost complex structures $J_{\pm N}^{M}$ ($J_{\pm}^{2} = -1$), then one can obtain a second set of supersymmetry transformations for X^{M} and $\psi_{\pm}^{M}$ [16, 17, 18, 19, 20]. This is achieved by making the replacement $\psi_{\pm}^{M} \to \psi_{\pm}^{(J)M} = J_{\pm N}^{M}\psi_{\pm}^{N}$ in the $N = 1$ supersymmetry transformations (2)–(4). The extended supersymmetry transformations are then given by

$$\delta_{\mp}^{(J)}X^{M} \;=\; \pm i\epsilon_{\mp}J_{\pm N}^{M}\psi_{\pm}^{N} \tag{20}$$

$$\delta_{\mp}^{(J)}\psi_{\pm}^{M} \;=\; \mp J_{\pm N}^{M}\partial_{\pm}X^{N}\epsilon_{\mp} \mp i(J_{\pm N}^{M}\partial_{L}J_{\pm K}^{N}J_{\pm P}^{L})\psi_{\pm}^{K}\epsilon_{\mp}\psi_{\pm}^{P} \tag{21}$$

$$\delta_{\mp}^{(J)}\psi_{\mp}^{M} \;=\; \mp i\psi_{\mp}^{N}\epsilon_{\mp}\Omega_{NK}^{\pm M}J_{\pm L}^{K}\psi_{\pm}^{L}. \tag{22}$$

Clearly, for the action (1) to be invariant under the extended supersymmetry transformations, it is sufficient that it is invariant under the replacement $\psi_{\pm} \to \psi_{\pm}^{(J)}$. This requires that the metric G_{MN} be Hermitian with respect to $J_{\pm N}^{M}$ and that the almost complex structures be covariantly constant with respect to the torsionful connections $\Omega_{NK}^{\pm M}$:

$$J_{\pm K}^{M}G_{MN}J_{\pm L}^{N} = G_{KL} \tag{23}$$

$$\nabla_{M}^{\pm}J_{\pm K}^{N} \equiv \partial_{M}J_{\pm K}^{N} + \Omega_{ML}^{\pm N}J_{\pm K}^{L} - J_{\pm L}^{N}\Omega_{MK}^{\pm L} = 0 \quad . \tag{24}$$

The above conditions ensure that the bilinear fermion terms and the four-fermion terms in the action (1) are separately invariant under $\psi_{\pm} \to \psi_{\pm}^{(J)}$. The two sets of supersymmetry transformations, (2)–(4) and (20)–(22), satisfy the usual $N = 2$ algebra, provided the Nijenhuis tensors corresponding to $J_{\pm}$ vanish and hence the almost complex structures are integrable. (For a discussion of the more general case where this is not true, see [26].) The above discussion can be easily generalized to the extension of $N = 1$ to $N = 4$ supersymmetry which requires the existence of three complex structures satisfying a quaternionic algebra. It is clear that the existence of an extended supersymmetry on the worldsheet is related to

the geometrical properties of the target manifold. In particular, eq. (24) restricts the holonomy of the target manifold. We will see that for the dual theory the situation is somewhat different. Our results in the following are independent of the details of the extended supersymmetry and hence the complex structures we consider could be either associated with an $N = 2$ or $N = 4$ supersymmetry.

The issue we want to address now is how T-duality affects the complex structures and therefore the extended worldsheet supersymmetry. Here, there are two possibilities: (i) $\mathcal{L}_K J = 0$ and (ii) $\mathcal{L}_K J \neq 0$. In the first case, which in our preferred coordinate system corresponds to θ-independent complex structures, the answer is known [21, 22]. In this case, the dual theory also admits almost complex structures $\widetilde{J}_\pm$ given by

$$\widetilde{J}_\pm = Q_\pm J_\pm Q_\pm^{-1}, \tag{25}$$

where $Q_\pm$ are defined in (6). It can be shown that $\widetilde{J}_\pm$ are covariantly constant and integrable [22]. Therefore, in this case the supersymmetry in the dual model is realized in the usual manner. The discussion of duality and supersymmetry in [17, 9, 27] falls in this category.

In the following, we concentrate on the case $\mathcal{L}_K J_\pm \neq 0$ which corresponds to θ-dependent complex structures. In this case, it can be easily checked that $\widetilde{J}_\pm(\theta, x^i)$, as given by (25), are no longer covariantly constant. This implies that the action (1) is not invariant under the corresponding supersymmetry transformations. Examples of this type of models were first encountered in [6]. Since duality is a symmetry of the underlying conformal field theory, one expects that supersymmetry survives though, in the dual model, it is not realized in the standard manner. In fact, the discussion in the previous section on the canonical approach to duality transformations indicates that in the dual theory the θ-dependent complex structures are replaced by non-local objects. To see this, the first step is to find the non-local objects which replace the complex structures $J(\theta, x^i)$ in the dual theory. This can easily be done by requiring the covariance under duality of the extended supersymmetry transformation (22). Let us again denote the objects dual to the complex structures by $\widetilde{J}_{\pm N}^M$. Then, using (13), (14) and (8), we obtain

$$\widetilde{J}_\pm([\widetilde{\theta}, x^i], x^i) = Q_\pm J_\pm(\theta[\widetilde{\theta}, x^i], x^i) Q_\pm^{-1}. \tag{26}$$

Here, $\theta[\widetilde{\theta}, x^i]$ is the usual notation for the functional dependence of θ on $\widetilde{\theta}$ and x^i with the explicit relation given by (14). It is clear that now $\widetilde{J}_\pm$ has a non-local dependence on the coordinates of the dual target space $\{\widetilde{X}^M\} = \{\widetilde{\theta}, \ x^i\}$. For θ-independent complex structures, (26) reduces to (25). Equation (26) is also in agreement with the prescription given in [5] for obtaining the non-local supersymmetry, except for the inclusion of the worldsheet fermions in the relation between θ and $\widetilde{\theta}$.

The extended supersymmetry transformations of the dual model can now be defined in the standard way: as the $N = 1$ transformations acting on $\widetilde{\psi}_\pm^{(J)} = \widetilde{J}_\pm \widetilde{\psi}_\pm$. Since, on shell, $\widetilde{\psi}_\pm^{(J)}$ is a local function of the worldsheet coordinates, the variation of the dual action under

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the above transformations can still be obtained without any complications. However, these transformations can no longer be written in the form (20)–(22) since the derivatives of the non-local objects $\widetilde{J}_\pm$ now have to be properly treated. The derivatives of $\widetilde{J}$ which still make sense are the partial derivatives with respect to x^i and the derivative with respect to the original θ coordinate, *i.e.* $\partial_i \widetilde{J}$ and $\partial_\theta \widetilde{J}$. We will therefore express all results in terms of these derivatives since the ordinary derivative of $\widetilde{J}$ with respect to $\widetilde{\theta}$ is not defined (it turns out that a functional $\widetilde{\theta}$-derivative is not the natural object to replace the ordinary θ-derivative in the dual theory). In particular, using (14), we can write

$$\partial_\pm \widetilde{J} = \partial_i \widetilde{J} \, \partial_\pm x^i + \left[(Q_\pm^{-1})^\theta_L \, \partial_\pm \widetilde{X}^L + i \, G_{\theta\theta}^{-1} \, F_\pm \right] \partial_\theta \widetilde{J}, \tag{27}$$

where $\widetilde{J}$ could be either $\widetilde{J}_+$ or $\widetilde{J}_-$.

We now investigate the conditions under which the action dual to (1) is invariant under the replacement $\widetilde{\psi}_\pm \to \widetilde{\psi}_\pm^{(J)} = \widetilde{J}_\pm \widetilde{\psi}_\pm$ and, therefore, under the corresponding extended supersymmetry transformations. For definiteness, let us focus on the transformations involving $\widetilde{J}_+$. Since $\widetilde{J}_+$ is not covariantly constant, the four-fermion terms and the terms bilinear in $\widetilde{\psi}_+$ appearing in the dual action are not separately invariant under the above replacement. We first calculate the variation of the four-fermion term in the dual theory. In (19), we can replace ψ_+ by $\psi_+^{(J)}$ since they both transform in the same way under duality. This gives the four-fermion term in the dual theory after the replacement $\widetilde{\psi}_+ \to \widetilde{\psi}_+^{(J)}$ as

$$\widetilde{\psi}_+^{(J)M} \widetilde{\psi}_+^{(J)N} \widetilde{\psi}_-^K \widetilde{\psi}_-^L \widetilde{R}_{MNKL}(\widetilde{\Omega}^-) = \psi_+^{(J)M} \psi_+^{(J)N} \psi_-^K \psi_-^L R_{MNKL}(\Omega^-) + 2 G_{\theta\theta}^{-1} F_+^{(J)} F_-. \tag{28}$$

Now, using the covariant constancy of J_+ in $F_+^{(J)}$, we get

$$F_+^{(J)} = F_+ + \psi_+^M \, G_{MN} \, J_{+L}^N \, \partial_\theta J_{+K}^L \, \psi_+^K. \tag{29}$$

Notice that $F_+^{(J)} - F_+$ is self-dual. Substituting this back in (28) and using (19), we obtain the variation of the four-fermion term as

$$\widetilde{\psi}_+^{(J)M} \widetilde{\psi}_+^{(J)N} \widetilde{\psi}_-^K \widetilde{\psi}_-^L \widetilde{R}_{MNKL}(\widetilde{\Omega}^-) \; - \; \widetilde{\psi}_+^M \widetilde{\psi}_+^N \widetilde{\psi}_-^K \widetilde{\psi}_-^L \widetilde{R}_{MNKL}(\widetilde{\Omega}^-)$$
$$= \; 2 \, \widetilde{\psi}_+^M \, \widetilde{G}_{MN} \widetilde{J}_{+L}^N \partial_\theta \widetilde{J}_{+K}^L \, \widetilde{\psi}_+^K \, G_{\theta\theta}^{-1} \, F_-. \tag{30}$$

To this, we add the variation under the replacement $\widetilde{\psi}_\pm \to \widetilde{J}_\pm \widetilde{\psi}_\pm$ of the terms bilinear in ψ_+. Setting the total variation to zero gives us the condition for the invariance of the dual theory under the non-local extended supersymmetry as

$$\partial_- \widetilde{J}_{+N}^M - i G_{\theta\theta}^{-1} \partial_\theta \widetilde{J}_{+N}^M F_- + \left(\widetilde{\Omega}_{KL}^{+M} \widetilde{J}_{+N}^L - \widetilde{J}_{+L}^M \widetilde{\Omega}_{KN}^{+L} \right) \partial_- \widetilde{X}^K = 0. \tag{31}$$

Using (27), the above equation (and the similar equation involving $\widetilde{J}_-$) can be written as conditions on $\widetilde{J}_\pm$:

$$\widetilde{G}_{\widetilde{\theta}\widetilde{\theta}} \partial_\theta \widetilde{J}_{\pm N}^M + \widetilde{\Omega}_{\widetilde{\theta}L}^{\pm M} \widetilde{J}_{\pm N}^L - \widetilde{J}_{\pm L}^M \widetilde{\Omega}_{\widetilde{\theta}N}^{\pm L} = 0. \tag{32}$$
$$\widetilde{\nabla}_i^\pm \widetilde{J}_{\pm N}^M \pm (\widetilde{G} \pm \widetilde{B})_{\widetilde{\theta}i} \, \partial_\theta \widetilde{J}_{\pm N}^M = 0$$

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The above equations generalize the condition of covariant constancy of complex structures to the case where the extended supersymmetry of the dual theory is realized non-locally. Notice that, to avoid any confusion, in eqs. (32) the index in the $\widetilde{\theta}$ direction has been explicitly labeled so. In the previous equations, where there is no risk of confusion, the index θ was used to represent either a θ or a $\widetilde{\theta}$ direction. As a consistency check, notice that eqs. (32) are compatible with the ones obtained directly from (26) by assuming that the J is covariantly constant.

Even when the extended supersymmetry becomes non-local under duality, the extended superconformal algebra remains unchanged. However, this algebra is now realized in terms of non-local supercharges and the representation becomes non-local. Such non-local representations in a class of conformal field theories were constructed in terms of parafermions in [28]. The relevance of these representations to the behaviour of supersymmetry under duality was discussed in [5]. There are several explicit examples known in which a part of the extended supersymmetry becomes non-local under duality [6, 5]. In all of these cases the original theory has an $N = 4$ supersymmetry such that, in each chiral sector, two out of the three complex structures are θ-dependent. As a result, after a duality transformation with respect to the θ-isometry, only an $N = 2$ supersymmetry is locally realized. However, it is clear that the non-localization of extended supersymmetry is a generic feature of θ-dependent complex structures and is therefore not necessarily restricted to $N = 4$ theories.

As discussed in the previous section (below eq.(15)), a special situation arises when $Q_- = 1_D$, so that the background fields G and B are self-dual. It was argued that, in this case, the non-local dependence on the dual coordinate can be removed by a chiral shift of θ, which is now a symmetry of the theory. For backgrounds of this kind, the covariant constancy of J_- implies that $\partial_\theta J_- = 0$, so that $\widetilde{J}_- = J_-$. On the other hand, J_+ can still have a θ-dependence and $Q_+ \neq 1_D$. Therefore, in general, $\widetilde{J}_+ \neq J_+$, although $\widetilde{J}_+$ is still local and covariantly constant, as can be seen from (32). An example in which this situation arises is the supersymmetric $SU(2) \times U(1)$ WZNW model which has extended $N = 4$ supersymmetry on the worldsheet. If we use the usual Euler parametrization for the $SU(2)$ group, then the resulting theory has a manifest chiral isometry with respect to which the backgrounds are self-dual. The coordinate θ is now conjugate to the $SU(2)$ Cartan generator, say, T_3. The complex structures are defined by their action on the Lie algebra at the identity and can be extended to the full group manifold using the left-invariant and the right-invariant one-forms. Out of the six complex structures (three in each chiral sector), only two in the left-moving sector are θ-dependent. Now, it can be verified that the transformations of the complex structures, as given by (26), are consistent with the well-known interpretation of duality as the automorphism $T_3 \to -T_3$, acting on the left-moving sector of the worldsheet theory. On the other hand, a duality with respect to the vector or the axial current leads to the $SU(2)/U(1) \times U(1)^2$ model in which part of the supersymmetry is non-locally realized [28, 5].

4 Implications for Target Space Supersymmetry

In this section we discuss the implications of the non-localization of extended worldsheet supersymmetry under T-duality for the associated target space supersymmetry.

In the low-energy limit of superstring theory, the massless background fields G_{MN}, B_{MN} and Φ, along with their superpartners, transform under two copies of $N = 1$ supersymmetry transformations. These have their origin in the independent left-moving and right-moving supersymmetries on the worldsheet. Since the backgrounds describe a vacuum configuration for the low-energy strings, the theory describing the fluctuations around these backgrounds will have unbroken supersymmetry if the backgrounds themselves are invariant. This can be achieved by setting to zero the fermionic backgrounds, along with their variations under supersymmetry. In the following, we explicitly consider one copy of the $N = 1$ supersymmetry transformations and, moreover, set the gravitino background Ψ_M and the dilatino background λ to zero. Let us denote the space-time supersymmetry transformation parameter by η which is a Majorana-Weyl spinor in $D = 10$. Then, setting to zero the variation under supersymmetry of the fermionic backgrounds gives:

$$\delta\Psi_M = \partial_M\eta + \frac{1}{4}\left(\omega_M^{AB} - \frac{1}{2}H_M{}^{AB}\right)\gamma_{AB}\,\eta = 0 \tag{33}$$

$$\delta\lambda = \gamma^M\partial_M\,\Phi\,\eta - \frac{1}{6}H_{MKN}\gamma^{MKN}\,\eta = 0. \tag{34}$$

Here, ω_M^{AB} is the spin connection, and the torsion term added to it has been explicitly exhibited. The indices A, B refer to the tangent space. Equation (33) defines a Killing spinor on the target space. It is known that if the Killing spinor η is independent of the coordinate θ, then the above equations are also satisfied in the dual theory [29]. In this case, the spinor η does not transform under duality. When $\partial_\theta\eta \neq 0$, the above equations are not satisfied in the dual theory. Also, in [6] an example was considered and it was argued that eq. (34) is not satisfied after a T-duality transformation. This suggests that in these cases, the target space supersymmetry of the dual theory is not realized in the conventional way.

The connection between the extended worldsheet supersymmetry and the target space supersymmetry, which is characterized by eqs. (33), (34), is well known [30, 31]. The complex structure associated with the worldsheet supersymmetry can be constructed in terms of η and (up to a constant factor) is given by

$$J_{+N}^M = \bar{\eta}\,\gamma_N^M\,\eta. \tag{35}$$

The Killing spinor condition then implies that $\nabla_M^+ J_{+N}^K = 0$. From (35), it is evident that if the vielbeins in $\gamma_N^M = e_A^M\,e_N^B\gamma_B^A$ are chosen to be independent of θ, then a θ-dependence of J implies a θ-dependence for η. The converse, however, is not always true and $\partial_\theta J = 0$ does not necessarily imply $\partial_\theta\eta = 0$.

Using (35), it is easy to see what happens to the target space supersymmetry parameter η under duality. There are three possibilities depending on who η and the associated complex structure J_+ depend on θ:

Case 1: Let us first consider the case $\partial_\theta \eta = 0$, which implies $\partial_\theta J_+ = 0$. Under duality, the transformation of J_+ is given by (25). As discussed below eq. (8), the T-dual of the (inverse) vielbein is not unique and is given by either $\widetilde{e}_+ = Q_+ e$ or $\widetilde{e}_- = Q_- e$. However, since the two are related by a local Lorentz transformation, we can choose either of them. If we choose $\widetilde{e}_+$ as the dual vielbein, then it is apparent that eq. (35) is also valid in the dual theory without transforming the spinor η. Since the dual complex structure is covariantly constant, it follows that η is also a Killing spinor for the dual theory[2]. For the second copy of target space supersymmetry, which is associated with J_-, it is natural to choose $\widetilde{e}_-$ as the dual vielbein. This can be re-expressed in terms of $\widetilde{e}_+$, using the local Lorentz transformation relating the two. From the analogue of eq. (35) for J_-, it then follows that this local Lorentz transformation can be absorbed in a redefinition of the Killing spinor associated with the second supersymmetry. Thus, the difference in the transformations of the two worldsheet chiral sectors under duality results in a redefinition of the spinors associated with the target space supersymmetry.

Case 2: Now, we address the issue of target space supersymmetry when the corresponding extended worldsheet supersymmetry becomes non-local under duality. As discussed in the previous section, this situation arises when $\partial_\theta J_\pm \neq 0$ and hence $\partial_\theta \eta \neq 0$. In this case, the non-local object $\widetilde{J}_+$, which is dual to the complex structure J_+, is given by eq. (26). For the dual vielbein, we again choose $\widetilde{e}_+$. Equation (35) then implies that in the dual theory there exists a non-local spinor $\widetilde{\eta}$ given by

$$\widetilde{\eta}([\widetilde{\theta}, x], x) = \eta(\theta[\widetilde{\theta}, x], x). \tag{36}$$

The analogue of the Killing spinor condition (33) for $\widetilde{\eta}$ can be obtained by substituting $\widetilde{J}_{+N}^M = \bar{\widetilde{\eta}} \widetilde{\gamma}_N^M \widetilde{\eta}$ in eq. (32) which is a generalization of the covariant constancy condition for $\widetilde{J}_+$. Corresponding to the θ-component and the i-components of the Killing spinor condition (33), we obtain the following two equations in the dual theory:

$$\widetilde{G}_{\widetilde{\theta}\widetilde{\theta}} \partial_\theta \widetilde{\eta} + \frac{1}{4} \left(\widetilde{\omega}_{\widetilde{\theta}}^{AB} - \frac{1}{2} \widetilde{H}_{\widetilde{\theta}}^{\ AB} \right) \gamma_{AB} \, \widetilde{\eta} = 0 \tag{37}$$

$$\partial_i \widetilde{\eta} + \frac{1}{4} \left(\widetilde{\omega}_i^{AB} - \frac{1}{2} \widetilde{H}_i^{\ AB} \right) \gamma_{AB} \, \widetilde{\eta} + (\widetilde{G} + \widetilde{B})_{\widetilde{\theta} i} \, \partial_\theta \widetilde{\eta} = 0.$$

The correctness of the above equations can also be checked directly by using the relation between η and $\widetilde{\eta}$ in (33). In the dual theory, we have not written down the equation that corresponds to the vanishing of the dilatino variation (34). This equation can be obtained by substituting the dual variables in (34). Equations (37) reduce to the usual Killing spinor

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[2]Since the same torsion term has been added both to the spin connection and to the affine connection, the vielbeins are still covariantly constant.

equation when either $\partial_\theta \widetilde{\eta} = 0$ or $Q_- = 1$. The later case, as discussed in the previous sections, corresponds to a self-dual theory with a chirally conserved current.

Case 3: The only other possibility is when the target space spinor η depends on the coordinate θ in such a way that the θ-dependences on the right hand side of (35) cancel out, giving rise to a θ-independent J. In this case, in the dual theory, the extended worldsheet supersymmetry is locally realized while the associated target space supersymmetry has a non-local realization. It turns out that the model considered in [6] and [29], in connection with the apparent violations of supersymmetry under T-duality, falls in this class. In the following we briefly describe this model in order to clarify its behaviour based on our approach. Consider the four-dimensional flat Euclidean space $\{X^1, X^2, X^3, X^4\}$. This space admits three complex structures J^a, $a = 1, 2, 3$, which satisfy a quaternionic algebra and the corresponding theory has $N = 4$ supersymmetry. When the metric is flat, one of the complex structures (say J^3) is in the canonical form. Now, let us choose polar coordinates in the $X^1 X^2$-plane: $\{X^1, X^2\} \rightarrow \{r, \theta\}$. Two of the complex structures, J^1 and J^2, develop a θ-dependence while J^3 and the metric are independent of θ. To keep the vielbeins also θ-independent, we have to perform a θ-dependent local Lorentz transformation. As is evident from (35), this Lorentz transformation can be absorbed in the target space spinors associated with J^a, thus making them θ-dependent. Now, under a T-duality transformation with respect to θ, all target space and worldsheet supersymmetries become non-local except for the worldsheet supersymmetry associated with J^3.

In some cases, such as the $SU(2) \times U(1)$ WZNW model, the non-local nature of the target space supersymmetry can also be interpreted in a somewhat different way. In this model there is a natural choice for the vielbeins in terms of the left-invariant or right-invariant one-forms. In this case, although the metric is still θ-independent, the vielbeins are not. The full θ-dependence of the complex structure in (35) is then contained in the vielbein and not in the spinor η. As a result, in the dual theory, $\widetilde{\eta}$ is local but the vielbein transforms into a non-local object. In this scenario, the dual target space possesses a supersymmetry that is defined not in a standard local Lorentz frame, but in a frame connected to the latter by a non-local rotation.

Since duality is a symmetry, one expects that the dual target space theory also admits some kind of supersymmetry with the non-local spinor $\widetilde{\eta}$ as the transformation parameter. Though the explicit form of this transformation is not known, in the following, we will argue that they are expected to be very different from the standard target space supersymmetry transformations. To see this, note that since the dual backgrounds are expressed in terms of the original ones, it follows that if the fermionic backgrounds in the original theory are set to zero, they will remain zero in the dual theory. This is consistent with the invariance of the dual backgrounds under the non-local supersymmetry. It is then reasonable to expect, in analogy with the local case, that the gravitino variation in the dual theory is proportional to the left-hand side of (37). The form of this transformation is clearly different from the usual supersymmetry transformation of the gravitino given by (33). In particular, due to its

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non-locality, this transformation makes sense only when the coordinates are restricted to the string worldsheet, and not at a generic space-time point. The existence of this transformation in the dual theory would not have been evident in the absence of the duality relation. Since the background fields are invariant under supersymmetry, the non-locality does not show up as long as we are looking only at the vacuum configurations. However, one expects that the modification of the supersymmetry transformations should have important consequences for the spectrum of fluctuations around these backgrounds which are the relevant quantum fields for the low-energy theory. In particular, it is unlikely that the dual theories are equivalent as quantum field theories.

As string theories, the equivalence under duality is a consequence of the existence of both momentum and winding modes associated with the compact coordinate θ. It is well known that under duality these modes are interchanged: The conserved momentum P_θ and the winding number L_θ associated with the compact coordinate θ (with non-trivial π_1) are given by $P_\theta = \int_0^{2\pi} d\sigma p_\theta$ and $L_\theta = \theta(\sigma = 2\pi) - \theta(\sigma = 0)$. Then, from the canonical transformations (9), it follows that $\widetilde{P}_\theta = -L_\theta$ and $\widetilde{L}_\theta = -P_\theta$. Since the momentum and winding modes are associated with the worldsheet coordinates τ and σ, respectively, their interchange under duality is the origin of the non-local relationship between θ and $\widetilde{\theta}$. This can be easily see when the backgrounds are flat and one can write $\theta = \theta_L + \theta_R$ whereas, $\widetilde{\theta} = \theta_L - \theta_R = \int d\sigma^+ \partial_+\theta - \int d\sigma_- \partial_-\theta$. As for the behaviour of supersymmetry, note that the parameter $\eta(\theta, x)$ is sensitive to the string momentum and winding modes associated with θ. The non-locality in the dual theory arises due to the fact that the the momentum and winding modes of the dual string enter $\widetilde{\eta}$ not through $\widetilde{\theta}$ (which would have resulted in a local spinor $\widetilde{\eta}(\widetilde{\theta})$), but through the original coordinate θ.

5   Summary and Discussions

In this section, we first summarize our results and then briefly discuss their generalization to $O(d, d)$ deformations. At the end, we discuss an apparent violation of worldsheet supersymmetry under S-duality transformations.

We have addressed the issue of the non-locality of extended worldsheet supersymmetry and the associated target space supersymmetry under T-duality transformations. This happens when the complex structure J associated with the extended supersymmetry has a dependence on the coordinate (say θ) with respect to which duality is performed. To study this issue systematically, we first generalized the implementation of a T-duality by a canonical transformation from the bosonic to the supersymmetric non-linear σ-model. Using this, we obtained the non-local object $\widetilde{J}$ which, in the dual theory, replaces the θ-dependent complex structure J. Similar to the complex structure, this non-local object defines the extended supersymmetry of the dual model in terms of its default $N = 1$ supersymmetry. The

extended supersymmetry of the dual model is thus realized non-locally. The non-locality is only in terms of the target space coordinates and, on shell, the theory is local in terms of the worldsheet coordinates. The invariance of the dual action under the non-local supersymmetry imposes some restrictions on $\widetilde{J}$, which are analogous to the covariant constancy condition for the complex structure in the usual realization of the supersymmetry. We then used the relation between the extended worldsheet supersymmetry and the target space supersymmetry of the original theory to argue that the dual target space admits supersymmetry transformations with a non-local spinor parameter. We also obtained the analogue of the Killing spinor equation which this non-local spinor satisfies. The analysis suggests that the action of the non-local supersymmetry on the backgrounds is different from the action of the standard target space supersymmetry. Thus, the supersymmetry of the dual theory has a non-standard realization on the spectrum of fluctuations around these backgrounds and the two theories are not equivalent as field theories. The equivalence as string theories is a consequence of the presence of winding (or winding like) modes in the string spectrum. The emergence of non-local effects in the low-energy theory is related to the momentum-winding interchange under T-duality. When the backgrounds are self-dual, the isometry with respect to which duality is performed becomes chiral. It was shown that in this case the non-locality can be removed by a chiral shift of the coordinate. In these situations, as expected, both the worldsheet and the target space supersymmetries remain local.

The mechanism by which the non-locality appears in the theory is not specific to duality transformations. On the contrary, it is common to all non-trivial $O(d, d)$ transformations which contain the T-duality transformations as a discrete subgroup [13, 14]: Let θ^m denote the d coordinates on which the background fields do not depend, and let p_m denote the corresponding conjugate momenta. If we define a $2d$ -dimensional vector Z as $Z^T = (-\theta'^T, p^T)$, then an $O(d, d)$ transformation can be implemented by the canonical transformation $\widetilde{Z} = \Omega Z$, where $\Omega \in O(d, d)$. Notice that these canonical transformations do not in general lead to equivalent quantum theories but rather correspond to the deformations of the original theory [33]. The transformation of the complex structures is again given by an equation similar to (26), with the difference that now $Q_{\pm}$ are the general matrices given in [22]. This shows that theories with extended supersymmetry, which have θ^m-dependent complex structures and also admit $O(d, d)$ deformations, actually correspond to very special points in a theory space where, generically, the extended supersymmetry has a non-local realization. An example of this is the space of deformations of the $SU(2) \times U(1)$ WZNW model with $N = 4$ worldsheet supersymmetry.

Recent work has revealed a close connection between T-duality and S-duality transformations, notably the fact that in some theories their roles get interchanged (see [34] and references therein). It is known that S-duality acts as an R-symmetry on the target space spinors [35]. However, there exists an example where the extended worldsheet supersymmetry is not preserved under S-duality (to be more precise, a one-parameter family of $SL(2, R)$ transformations). This example, which involves an intertwining of S- and T-dualities, was considered in [6] in connection with the apparent supersymmetry violations of T-duality and

was also described in the previous section (under Case 3). There we saw that in this model, after a T-duality, the target space supersymmetries are non-locally realized though we are still left with one locally realized worldsheet supersymmetry. A combination of S-T transformations on this background again leads to a pure gravitational background with a metric that is Ricci-flat but not hyper-Kähler. This means that the metric could not be Kähler and therefore does not admit a covariantly constant complex structure. This in turn shows that the surviving $N = 2$ supersymmetry is no longer manifest after the S- and T-duality transformations. There are two possibilities for this to happen. One possibility is that the surviving complex structure develops a θ-dependence as a result of the S-duality and therefore becomes non-local after the final T-duality. The other possibility is that the complex structure does not survive the S-duality. An explicit calculation shows that it is the second possibility that actually occurs. We therefore have a situation where S-duality destroys a complex structure. Note that there is no contradiction with the space-time supersymmetry since there are no locally realized target space supersymmetries. Unlike the case of T-duality, it is not clear what the origin of this phenomenon is and what happens to the supersymmetry associated with this complex structure.

Acknowledgements

Note Added

The problem of duality and supersymmetry has recently been studied in [36] from a different point of view.

References

[1] See for example, M. B. Green, J. H. Schwarz and E. Witten, *Superstring Theory*, vol. II, Cambridge University Press (1987).

[2] E. Kiritsis, Nucl. Phys. **B405** (1993) 109, (hep-th/9302033).

[3] E. Álvarez, L. Álvarez-Gaumé and Y. Lozano, Nucl. Phys. **415** (1994) 71, (hep-th/9309039).

[4] E. Álvarez, L. Álvarez-Gaumé and Y. Lozano, Phys. Lett. **B336** (1994) 183, (hep-th/9406206).

[5] I. Bakas and K. Sfetsos, *T-duality and World-sheet Supersymmetry*, CERN-TH/95-16, THU-95/01, (hep-th/9502065).

[6] I. Bakas, Phys. Lett. **B343** (1995) 103, (hep-th/9410104).

[7] T. Buscher, Phys. Lett. **159B** (1985) 127; **194B** (1987) 59; **201B** (1988) 466.

[8] E. Kiritsis, Mod. Phys. Lett. **A6** (1991) 287.

[9] M. Roček and E. Verlinde, Nucl. Phys. **B373** (1992) 630, (hep-th/9110053).

[10] A. Giveon and M. Roček, Nucl. Phys. **B380** (1992) 128, (hep-th/9112070).

[11] A. Giveon, M. Porrati and E. Rabinovici, Phys. Rep. **244**(1994) 77, (hepth/9401139).

[12] E. Álvarez, L. Álvarez-Gaumé and Y. Lozano, *An Introduction to T-duality in String Theory*, CERN-TH/7486-94, FTUAM-94/26, (hep-th/9410237).

[13] A. Giveon, E. Rabinovici and G. Veneziano, Nucl. Phys. **B322** (1989) 167.

[14] K. Meissner and G. Veneziano, Phys. Lett. **B267** (1991) 33.

[15] C. Klimčik and P. Ševera, *Strings in Spacetime Cotangent Bundle and T-duality*, CERN-TH/7490/94, (hep-th/9411003).

[16] B. Zumino, Phys. Lett. **B88** (1979) 203;
L. Álvarez-Gaumé and D. Z. Freedman, Phys. Rev. **D22** (1980) 846; Commun. Math. Phys. **80** (1981) 443.
L. Álvarez-Gaumé, D. Z. Freedman and S. Mukhi, Ann. Phys. **134** (1981) 85.

[17] S. Gates, C.M. Hull and M. Roček, Nucl. Phys. **B248** (1984) 157.

[18] C.M. Hull and E. Witten, Phys. Lett. **160B** (1985) 398.

[19] P. Howe and G. Papadopoulos, Nucl. Phys. **B289**(1987) 264; Class. Quant. Grav. **5** (1988) 1647.

[20] A. Sen, Nucl. Phys. **B278** (1986) 289.

[21] I. T. Ivanov, B. Kim and M. Roček, Phys. Lett. **B343** (1995) 133, (hep-th/9406063);
B. Kim, Phys. Lett. **B335** (1994) 51, (hep-th/9406150).

[22] S. F. Hassan, *O(d,d;R) Transformations of Complex Structures and Extended World-sheet Supersymmetry*, TIFR/TH/94-26, (hep-th/9408060).

[23] A. Sevrin, W. Troost, A. Van Proeyen and Ph. Spindel, Nucl. Phys. **B308** (1988) 662; **B311** (1988/89) 465.

[24] T. Curtright and C. Zachos, Phys. Rev. **D 49** (1994) 5408; *The Paradigm of Pseudodual Chiral Models*, ANL-HEP-CP/94-33, hep-th/9407044; *Canonical Nonabelian Dual Transformations in Supersymmetric Field Theories*, MIAMI-TH/1-95,hep-th/9502026.

[25] W. Siegel, Phys. Rev. **D48** (1993) 2826, (hep-th/9305073);
Ashok Das and Jnanadeva Maharana, Mod. Phys. Lett. **A9** (1994) 1361, (hep-th/9401147);
A. Ali, *Duality Invariant Superstring Actions*, IP-BBSR-94-32, (hep-th/9406189).

[26] G. Papadopoulos, *(2,0)-Supersymmetric Sigma Models and Almost Complex Structures*, DAMTP-R/95/3, (hep-th/9503063).

[27] E. Kiritsis, C. Kounnas and D. Lüst, Int. J. Mod. Phys. **A9** (1994) 1361, (hepth/9308124).

[28] C. Kounnas, Phys. Lett. **B321** (1994) 26;
I. Antoniadis, S. Ferrara and C. Kounnas, Nucl. Phys. **B421** (1994) 343.

[29] E. Bergshoeff, R. Kallosh and T. Ortin, Phys. Rev. **D51** (1995) 3009, (hep-th/9410230).

[30] P. Candelas, G. Horowitz, A. Strominger and E. Witten, Nucl. Phys. **B258** (1985) 46.

[31] A. Strominger, Nucl. Phys. **B274** (1986) 253.

[32] A. Sen, Phys. Lett. **B271** (1991) 295;
S. F. Hassan and A. Sen, Nucl. Phys. **B375** (1992) 103, (hep-th/9109038).

[33] S. F. Hassan and A. Sen, Nucl. Phys. **B405** (1993) 143, (hep-th/9210121);
M. Henningson and C. Nappi, Phys. Rev. **D48** (1993) 861, (hep-th/9301005).

[34] A. Sen, *String-String Duality Conjecture in Six Dimensions and Charged Solitonic Strings*, TIFR-TH/95-16, (hep-th/9504027);
J. A. Harvey and A. Strominger, *The Heterotic String is a Soliton*, EFI/95-16, (hep-th/9504047).

[35] T. Ortin, *Duality Versus Supersymmetry*, QMW-PH/94-35, hep-th/9410069.

[36] E. Álvarez, L. Álvarez-Gaumé and I. Bakas, *T-Duality and Space-Time Supersymmetry*, CERN-TH/95-201, FTUAM/95-101, hep-th/9507112.